Newton's Law of Gravitation

Gravitational Acceleration

Newton's law of universal gravitation states that there is a force of attraction F between two particles with masses m_1 and m_2 separated by a distance r that is represented by the relationship

$$F = G\frac{m_1 m_2}{r^2} \qquad (6.1)$$

where G is the universal gravitation constant. The value of G first was determined in 1798 by Lord Cavendish; the present value, which was determined in 1942, is equal to

$$6.6732 \times 10^{-11}\ \text{nt} \cdot \text{m}^2/\text{kg}^2 \quad \text{in SI units}$$

or

$$6.6732 \times 10^{-8}\ \text{dyne} \cdot \text{cm}^2/\text{g}^2 \quad \text{in cgs units}$$

Newton's Law for Object at Earth's Surface

If we assume that the Earth is spherical, the force exerted by the Earth on a spherical body with mass m resting on the Earth's surface is

$$F = G\frac{mM}{R^2} \qquad (6.2)$$

where M is the Earth's mass and R is its radius. This also assumes that density varies only with distance from the center of the Earth (that the Earth is spherically symmetric) and that R is large compared to the size of the object. Values that we will use in future calculations include

Earth radius (at equator): 6.367×10^8 cm,

Earth radius (at 45° latitude): 6.378×10^8 cm, and

Earth mass: 5.976×10^{27} g.

Force also is given by Newton's second law of motion, which states that

$$F = ma \qquad (6.3)$$

Definition of "little" g Gravitational Acceleration

$$F = mg = G\frac{mM}{R^2}$$

and

$$g = \frac{GM}{R^2}. \qquad (6.4)$$

The dimensions of g are L/M², which are expressed as m/s² (SI) or cm/s² (cgs). In geophysics the normal unit of gravitational acceleration is the Gal (in honor of Galileo), which is 1 cm/s². Because the variations in gravitational acceleration g (or *gravity* for short) in which we are interested are so small, we often use the milliGal (1 mGal = 0.001 Gal) for exploration purposes. The gravity unit *gu* often is used instead of the mGal. It is equal to 0.000001 m/s², which is 0.1 mGal.

Measuring Gravity – Ideal Pendulum

Relative Measurements Using a Pendulum

The classic instrument for measuring both absolute and relative gravity is the pendulum. An ideal, simple pendulum suspends a material point that is dimensionless from a massless string that does not stretch and is perfectly flexible. For such a device the period T is related to the length of the string l and gravitational acceleration by the relationship

$$T = 2\pi\sqrt{\frac{l}{g}} \,. \tag{6.5}$$

Measuring Gravity – Real Pendulum

Such an ideal pendulum cannot be constructed, as should be obvious from its description. For a real pendulum Equation 6.5 becomes

$$T = 2\pi\sqrt{\frac{K}{g}} \tag{6.6}$$

where K is a constant that represents the characteristics of a particular pendulum system. If we are able to determine T and K accurately, then we have a measured or observed value for gravity, which we refer to as g_{obs}. Unfortunately, K cannot be determined accurately, and absolute values measured in this fashion have an accuracy of no better than 1 mGal and often are not that good. In routine surveys we need to know g_{obs} to at least 0.1 mGal.

Measuring Gravity – Relative Measurements

However, all is not lost. If we use the same pendulum at two observation sites, say x and y, then we have

$$g_{obs_x} = \frac{4\pi^2 K}{T_x^2} \quad \text{and} \quad g_{obs_y} = \frac{4\pi^2 K}{T_y^2} \,. \tag{6.7}$$

Both values will be inaccurate due to the imprecision of K; but because the same value of K enters both measurements, the gravity values will vary from the true value of gravity by the same factor. Therefore, we can use them to determine the relative difference in gravity (Δg) by accurately measuring T at each site as

$$\Delta g = g_{obs_x} - g_{obs_y} \,. \tag{6.8}$$

Precision is on the order of 0.1 mGal; however, pendulum-based instruments not only are bulky but require substantial amounts of time to acquire a measurement. They have been superseded by the gravimeter, which is very portable. Reading times are short, and precision is on the order of 0.01 mGal.

Lacoste-Romberg Gravimeter Hooke's Law (Strain proportional to Stress)

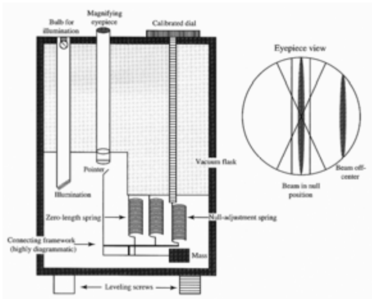
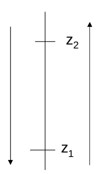


FIGURE 6.1 A highly diagrammatic representation of the interior of a Worden gravimeter.

Absolute Gravity Measurements

- Amount of time it takes an object to fall
- $z = 0.5\, g\, t^2$
- $g = 8(z_2-z_1)/[(t_4-t_1)^2 - (t_3-t_2)^2]$



Latitude Correction Earth is not a perfect sphere (see Table 6.1)

- $g_n = g_e(1 + A\sin^2 \phi - B \sin^4 \phi)$ cm/s^2
- $g_n = 978.03185(1 + 0.005278895 \sin^2 \phi - 0.000023462 \sin^4 \phi)$ cm/s^2

978 cm/s^2 = 978 Gals = 978,000 mGals

Free Air Correction

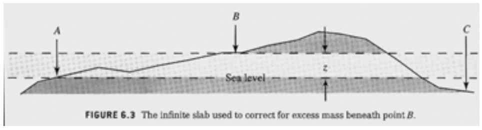


FIGURE 6.3 The infinite slab used to correct for excess mass beneath point *B*.

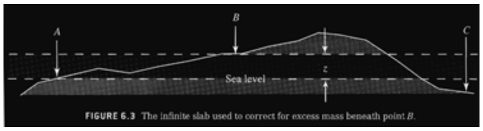


FIGURE 6.3 The infinite slab used to correct for excess mass beneath point *B*.

Elevation Correction

- $dg/dz = dg/dR = -2\ (GM/R^3) = -g(2/R)$
- $dg/dz = -0.3086 - 0.00023 \cos 2\phi$
  $+ 0.00000002z$
- $dg/dz = -0.3086$ mGal/m
- $\Delta g_{FA} = g_{obs} - g_n + FA_{corr}$

Simple Bouguer Correction

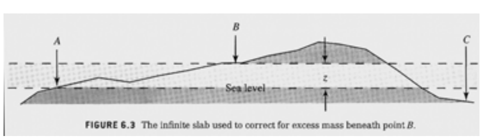


FIGURE 6.3 The infinite slab used to correct for excess mass beneath point *B*.

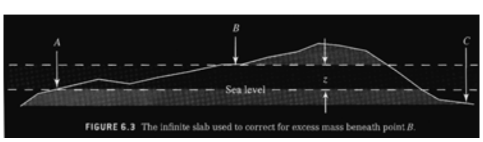


FIGURE 6.3 The infinite slab used to correct for excess mass beneath point *B*.

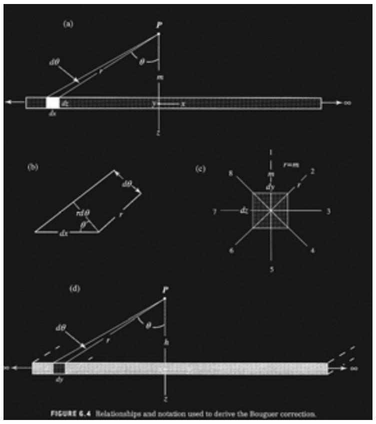


FIGURE 6.4 Relationships and notation used to derive the Bouguer correction.

Gravity Due to an Infinite Rod

The attraction of a thin rod is the sum of all these components, so we write

$$g = \int_{x=-\infty}^{x=\infty} \frac{G(\rho dx dy dz)}{r^2} \cos\theta . \qquad (6.18)$$

Before evaluating the integral, we note that $r = m/\cos\theta$. Also (see Figure 6.4(b)), because the length of arc for a small angle $d\theta$ equals the radius r times the change in angle $d\theta$, it is straightforward to demonstrate that $dx = r\, d\theta/\cos\theta$. Using these relationships, we show that

$$\frac{G(\rho dx dy dz)}{r^2}\cos\theta = \frac{G(\rho dy dz) r d\theta \cos\theta (\cos\theta)^2}{m^2 \cos\theta} = \frac{G(\rho dy dz) m d\theta \cos\theta (\cos\theta)^2}{m^2 \cos\theta \cos\theta} = \frac{G(\rho dy dz)}{m} \cos\theta\, d\theta .$$

Using this result, we adjust Equation 6.18 so that

$$g = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{G(\rho dy dz)}{m} \cos\theta\, d\theta \qquad (6.19)$$

which evaluates to

$$g = \frac{2G\rho dy dz}{m} . \qquad (6.20)$$

Equation 6.20 gives us g at P over a thin rod as illustrated in Figure 6.4(a).

Gravity Due to an Infinite Slab

The next step in our derivation is to sweep out a thin sheet using a thin rod (that is, use Equation 6.20 to calculate g over a thin sheet). Figure 6.4(c) is an end view of our thin rod. If lines 1, 2, 3, 4, 5, 6, 7, and 8 are of equal length, then g is the same at the end of every line. Line 1 is equivalent to m in our original diagram, and we will refer to line 2 as r. Clearly $m = r$ and g is the same at positions 1 and 2. This gives us the same view of the thin rod as we see in Figure 6.4(d), where line 2 now is labeled as r. Because we know g due to the thin rod (Equation 6.20), we use this to write

$$\Delta g_r = \frac{2G(\rho dy dz)}{r} . \qquad (6.21)$$

The next several steps mimic exactly those that we just listed, so following the same procedure we can say that

$$\Delta g_z = \frac{2G(\rho dy dz)}{r} \cos\theta \qquad (6.22)$$

$$g = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2G(\rho dz) d\theta \qquad (6.23)$$

and

$$g = 2\pi G(\rho dz). \qquad (6.24)$$

TABLE 6.2 EXAMPLE OF GRAVITY REDUCTION

Observed gravity	980658.67	Observed gravity	**980658.67**
Normal gravity	980674.39	Latitude (ϕ)	**45.62**
Free-air correction	30.93	Elevation (m)	**100.24**
Bouguer correction	11.22	Bouguer density (g/cm^3)	**2.67**
Free-air anomaly	15.22		
Bouguer anomaly	4.00		
Elevation error (m)	**0.33**	Latitude error (ϕ)	**0.01**
Bouguer anomaly error	0.06	Bouguer anomaly error	0.90

(All gravity values are in milliGals.)

Complete Bouguer
(Terrain Correction)

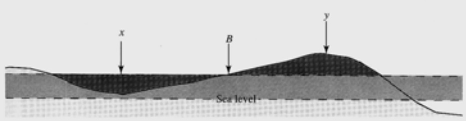


FIGURE 6.5 An illustration of terrain corrections required at regions *x* and *y* due to the oversimplified nature of the Bouguer correction applied at point *B*.

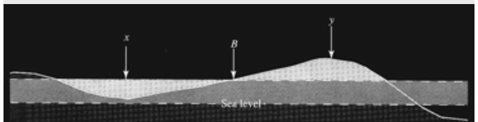


FIGURE 6.5 An illustration of terrain corrections required at regions *x* and *y* due to the oversimplified nature of the Bouguer correction applied at point *B*.

Cylindrical Ring of Mass

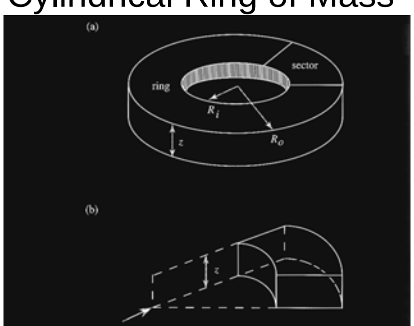


FIGURE 6.6 The basic approach to representing masses for terrain corrections. (a) A cylindrical ring with inner radius R_i, outer radius R_o, and thickness z. A section of the ring is referred to as a *sector*. (b) A sector represents a topographic feature above the level of a gravity observation point.

Sum of Pieces of Ring

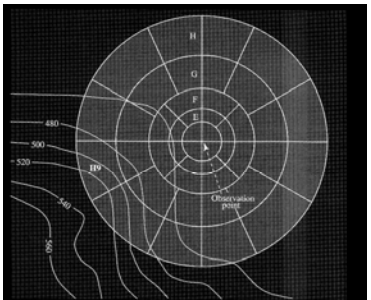


FIGURE 6.7 A terrain correction template placed on a topographic map with the center at a gravity observation point. Most contours are not shown for clarity. Only zones E, F, G, and H are illustrated.

Gravity Corrections (SI units)

$$\Delta g_B = g_{obs} - g_n + FA_{corr} - B_{corr}. \qquad (6.25)$$

It can be confusing to remember whether the Bouguer and free-air corrections should be added or subtracted, so it is best to record elevation differences from a datum as positive when above the datum and negative when below the datum. If this procedure is followed, the differences can be substituted directly into Equation 6.26, which is an expanded version of Equation 6.25, and the correction always will be in the proper sense.

$$\Delta g_B = (g_{obs} - g_n + 0.3086z - 0.04193\rho z)\ \text{mGal} \qquad (6.26)$$

Note that the units in Equation 6.26 are in milliGals, so g_{obs} and g_n also must be in milliGals.

Gravity Corrections (English units)

The majority of topographic maps in the United States still present elevations in feet, so many explorationists find it more convenient to retain feet as a unit when correcting gravity readings. In this case the proper form of the Bouguer anomaly equation becomes

$$\Delta g_B = (g_{obs} - g_n + 0.09406z - 0.01278\rho z)\ \text{mGal}. \qquad (6.27)$$

Because Bouguer anomaly values are the ones most often used for interpretation in gravity surveys, Equation 6.26 is presented in its complete expanded form as Equation 6.28. This incorporates the GRS67 formula and the free-air and Bouguer corrections.

$$\Delta g_B = \left\{ \begin{matrix} g_{obs} - [978031.85(1 + 0.005278895\sin^2\phi + 0.000023462\sin^4\phi)] + \\ [(0.3086 - 0.04193\rho)z] \end{matrix} \right\} \text{mGal} \qquad (6.28)$$

Lacoste-Romberg Gravimeter

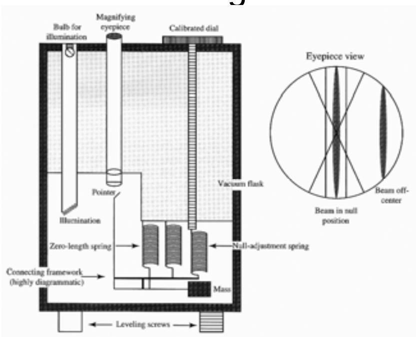


FIGURE 6.1 A highly diagrammatic representation of the interior of a Worden gravimeter.

Drift Correction

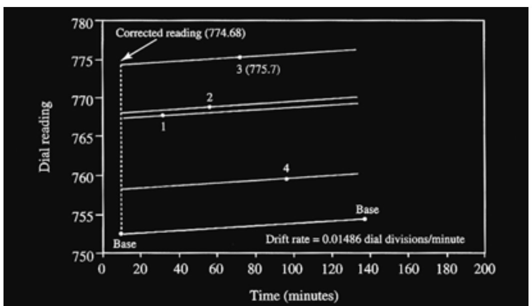


FIGURE 6.8 A typical drift curve for gravity observations at base and stations 1, 2, 3, and 4.

Ocean tides are the result of the gravitational attraction of the moon and sun on the ocean. The tides formed by the moon are the **lunar tides**, and those formed by the sun are the **solar tides**.

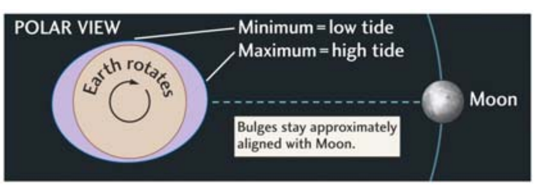


Fig. 20.3

Earth Tide Correction

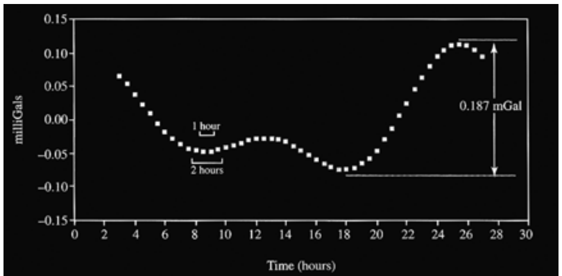


FIGURE 6.9 Variation of gravity values due to tidal effects. This is a representative curve for middle latitudes.

Drift and Tide Corrections

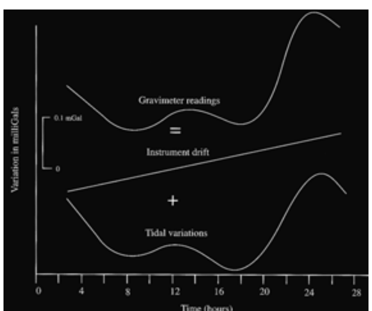


FIGURE 6.10 An illustration of how gravity values measured at a stationary position would vary due to a combination of tidal effects and linear instrument drift.

Repeat Measurements

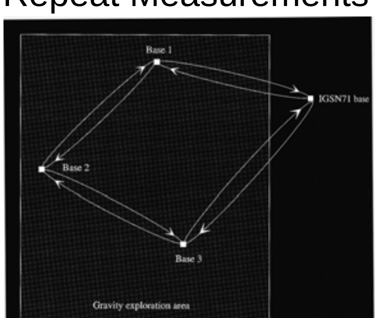


FIGURE 6.11 Establishing absolute gravity stations in an exploration area by referring them to an IGSN71 station and following a looping procedure.

Repeat Measurements

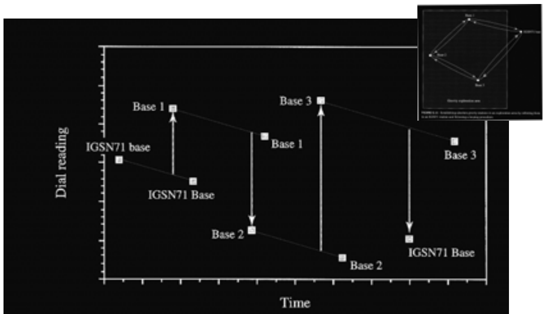


FIGURE 6.12 Drift curves resulting from the process illustrated in Figure 6.11.

Densities Used in Analysis (source of error)

TABLE 6.5 DENSITIES OF COMMON GEOLOGIC MATERIALS

Material	Density (g/cm^3)
Quartz	2.65
Feldspars	2.5–2.7
Micas	2.8
Calcite	2.72
Clay minerals	2.5–2.8
Unconsolidated sediments	1.7–2.3
Sandstones	2.0–2.6
Shales	2.0–2.7
Limestones	2.5–2.8
Dolomites	2.3–2.9
Granitic rocks	2.5–2.8
Basaltic rocks	2.7–3.1
Metamorphic rocks	2.6–3.0

Density of mineral:	**2.7**
Percentage porosity:	**50**
Density of saturated material:	1.85